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Packet Radio Temporary Note #178

AN APPROXIMATE ANALYTICAL MODEL FOR INITIALIZATION OF SINGLE HOP PACKET RADIO NETWORKS

May 19, 1976

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1. INTRODUCTION

Independent of the status of a network (stationary, versus mobile) and of the transmission discipline (packetized, reserved, etc.), a network employing radio links must be initialized before it can be used to transfer data from one point to another.

In a hardwired network, we know by construction, whether there is a link between two repeaters R_i and R_j; such link would be characterized by the existence of a cable or collection of cables between the two nodes.

In a radio network where we think of the repeaters as having been distributed randomly over some region, we have no a priori knowledge of the links between repeaters, and more generally, of the overall connectivity structure (unless the region under discussion is completely contained within the area of coverage of the repeater).

The objective of initialization is thus the detection of repeaters, the determination of hierarchical labels for repeaters, and the verification of links on paths associated with labels.

Initialization algorithms and labeling algorithms for stationary packet radio networks have been derived and discussed in detail in the literature; such algorithms with minor adjustments are applicable to mobile packet radio networks, as shown elsewhere. In real implementation, we have witnessed the integration of the initialization and monitoring tasks; however, in this note we concentrate on the initialization facet alone.

The basic initialization tools are:

ROP - Repeater On Packet

LLP - Label Packet

The ROP's are issued by (uninitialized) repeaters and contain pertinent repeater information; the LLP's are issued by the station(s) and contain labeling information.

A major issue related to initialization is the determination of the time required to complete the task. Simulation studies indicate that for a 60 repeater network, such time is approximately 2 seconds.

In this note we derive analytical models for the initialization procedure. Fairly realistic models are set up at first; however, to obtain closed form solutions we must assume worst case values for parameters, obtaining in conjunction, worst case values (or upper bounds) for initialization.

Due to such worst case approach to the analysis, comparisons between various models should be restricted to cases where the initialization times differ by some reasonably large value; smaller increments may indicate trends but this is not always the case; in fact on occasions the models may be insensitive and may not produce the expected differentiation.

2. MODEL

2.1 Single Hop Model Without Terminal Traffic

Figure 1 depicts the network under scrutiny. The following assumptions are made:

ASSUMPTIONS:

1. Single station with m repeaters, physically at 1-hop.

2. No repeater accepts ROP's from another repeater.

3. Each repeater can interfere with I neighbors (in addition to itself).

4. Common channel, slotted ALOHA.

5. Protocol.

   a. Each repeater broadcasts ROP's, such that the number of slots between any two transmissions is uniformly distributed on $[0,2\Omega]$. The ROP is transmitted without reference to the Active ACK (see 5.h below) whose effect is disregarded here.

   b. If a label is received by the repeater, it forwards one ETE ACK to the station and it halts its ROP emission except as in (c) below.

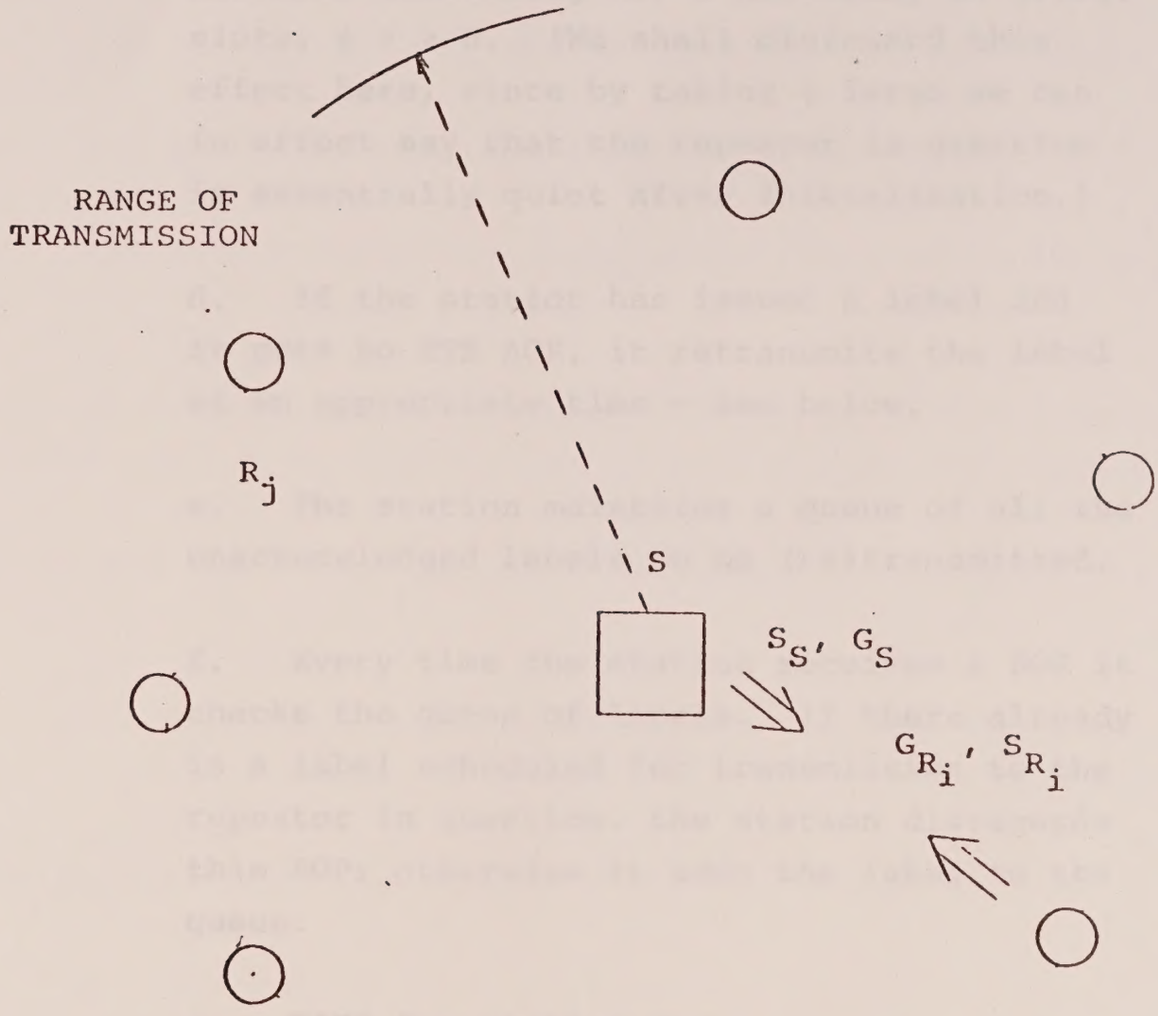

FIGURE 1: 1-HOP NETWORK

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c. If a repeater has received a label, it forwards monitoring ROP's uniformly on $[0,2\phi]$ slots, $\phi > > \Omega$. [We shall disregard this effect here, since by taking ϕ large we can in effect say that the repeater in question is essentially quiet after initialization.]

d. If the station has issued a label and it gets no ETE ACK, it retransmits the label at an appropriate time - see below.

e. The station maintains a queue of all the unacknowledged labels to be (re)transmitted.

f. Every time the station receives a ROP it checks the queue of labels. If there already is a label scheduled for transmission to the repeater in question, the station disregards this ROP; otherwise it adds the label to the queue.

g. FIFO for station queue.

h. When the station receives a ROP from R_i it issues an Active HBH ACK. [We shall disregard this process here, since, as it will be indicated later approximately less than 2.1 m such Active ACK packets would be issued in the process, and this traffic is negligible in comparison to the overall initialization traffic].

2.2 Machinery

Let $R_1, R_2, \ldots, R_m$ be the repeaters in question.

Let A_i = Time when the first ROP from R_i reaches S.

Let $Q(t)$ = Those repeaters whose label is in the station queue at time t.

$Q(t)$ represents those repeaters such that a ROP has been received by the station by time t, but no ETE ACK has been received.

A_i is the time when the label for i enters the queue for the first time.

Let B_i = Time when the label for i is deleted from the queue (namely it is received by R_i and acknowledged).

Definition: By initialization time we mean:

$$\mathcal{I} = \max_i E(B_i)$$

As an alternative one could look at

$$\mathcal{J} = E\left(\max_i B_i\right)$$

This is investigated cursorily in Appendix C.

Let $\theta_t = |Q(t)|$ = Number of labels to be issued at time t. We call θ_t the cycle length at t.

We now expand assumption 5d:

(5d') At the end of each cycle, say t*, S scans Q(t*); the labels found in the queue are then scheduled for transmission with an intertransmission delay distributed U[0,2w].

The remainder of this note is dedicated to expected value computations; hence we say that, on the average, one ROP every Ω slots is issued by a non-initialized R_i, and one LLP every w slots is broadcasted by S. With this in mind, we see that after S sends a label L_i to R_i, it times out for $\theta_{t^*} \cdot w$ slots on the average before forwarding a new label to R_i.

Observe that if t_{j-1} is the expected end of the j-1 cycle, then

$$t_j = t_{j-1} + \theta_{t_{j-1}} w$$

Let q_R and q_S be the probability of successful transmission on the repeater to station hop and on the station to repeater hop.

OBJECTIVES:

1. Determine q_R, q_S as a function of m, I, Ω, w.

2. Determine $\bar{t}$, given (1).

ANALYSIS:

The phenomenon is complicated. The flow chart of Figure 2 shows the steps involved in initialization.

Consider (for simplicity) a single R_i. It is broadcasting ROP's. When a ROP reaches S, the station tries to send a label to R_i. This may require several retransmissions by S (in the meantime R_i has issued more ROP's). When R_i finally receives a label it stops issuing the ROP's and it issues one ETE ACK. If the ETE ACK is received by S, R_i is eliminated from the queue; otherwise, S retransmits the label until it is received and acknowledged by R_i.

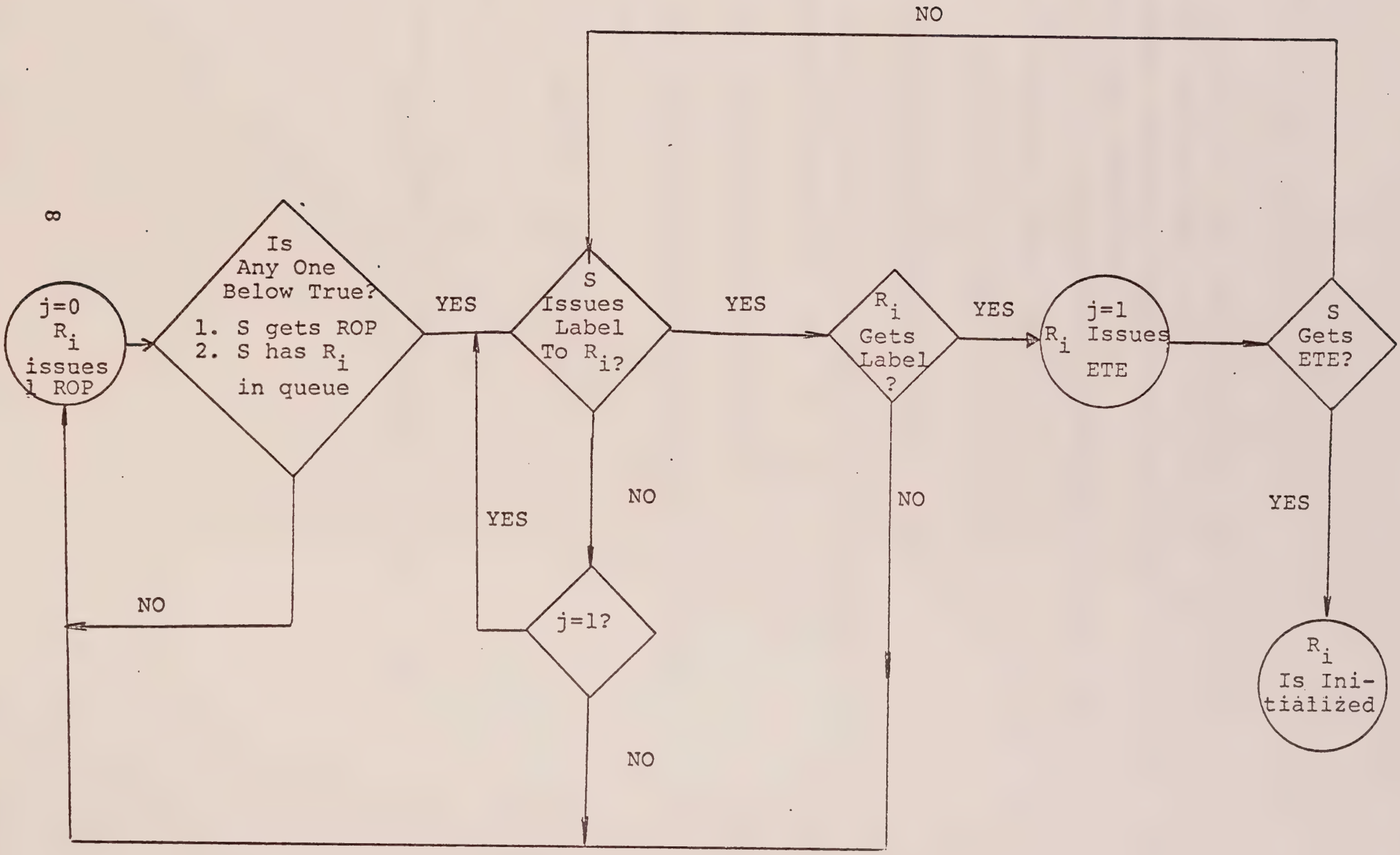

FIGURE 2: INITIALIZATION FLOW CHART

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2.3 Determination Of q_R and q_S

Let S_R, G_R be the total successful and total transmission rate, respectively, from the repeaters to the station. Let S_S, G_S be the total successful and total transmission rate, respectively, from the station to the repeaters.

Let S_{R_i}, G_{R_i} be the respective rates from R_i to S. Let S_{S,R_i}, G_{S,R_i} be the respective rates from S to R_i.

We make the following simplifying assumptions:

$$S_{R_i} = S_R/m; G_{R_i} = G_R/m; S_{S,R_i} = S_S/m; G_{S,R_i} = G_S/m$$

Consider a specific slot in the ALOHA broadcasting procedure.

Repeater R_i is idle if:

1. The station is not broadcasting.

2. The neighbors interfering with R_i are not broadcasting.

3. R_i itself is not broadcasting.

Thus,

$$\text{Prob}(R_i \text{ is idle}) = (1-G_S) \prod_{R_j \epsilon V_{R_i}} (1-G_{R_j}) \; (1-G_{R_i})$$

where,

$$V_{R_i} = \{R_j | d(R_i,R_j) < \mu,\ \mu = \text{range of repeaters}\} - R_i$$

As per assumption 3 above, $|V_{R_i}| = I$, $\forall_i$; also we postulated that $G_{R_i} = G_R/m$; thus

$$\text{Prob}(R_i \text{ is idle}) = (1-G_S)(1-\frac{G_R}{m})^{I+1}$$

The station is idle if

1. The station is not broadcasting;

2. No repeater is broadcasting.

Thus,

$$\text{Prob}(S \text{ is idle}) = (1-G_S)\prod_{i=1}^{m}(1-G_{R_i}) = (1-G_S)(1-\frac{G_R}{m})^m$$

Using the above, we can compute the expected throughput on the two links.

$$S_R = \sum_i (\text{traffic put out by } R_i)\ \text{Prob}(S \text{ is not disturbed by any repeater except } R_i, \text{ and is idle itself})$$

$$= \sum_i \frac{G_R}{m}(1-\frac{G_R}{m})^{m-1}(1-G_S) = G_R(1-\frac{G_R}{m})^{m-1}(1-G_S)$$

$$S_S = \sum_i (\text{traffic put out by S to } R_i)\ \text{Prob}(R_i \text{ is not disturbed by any device except S})$$

$$= \sum_i \frac{G_S}{m}(1-\frac{G_R}{m})^{I+1} = G_S(1-\frac{G_R}{m})^{I+1}$$

Now

$$q_S = \frac{S_S}{G_S} = \left(1-\frac{G_R}{m}\right)^{I+1}$$

$$q_R = \frac{S_R}{G_R} = \left(1-\frac{G_R}{m}\right)^{m-1}(1-G_S)$$

If $G_R = \rho(m)m$ and $G_R \to a$ monotonically from below as $m \to \infty$,

$G_S = \sigma(m)m$ and $G_S \to b$ monotonically from below as $m \to \infty$,

$I = \lambda m$

then

$$q_R = (1-\rho(m))^{m-1}\ (1-\sigma(m)m)$$

$$q_S = (1-\rho(m))^{\lambda m + 1}$$

The following relations hold

a. $q_R \downarrow$ as $m \uparrow$

b. $\lim_{m\to\infty} q_R = e^{-a}(1-b)$

c. $q_s \downarrow$ as $m \uparrow$

d. $\lim_{m\to\infty} q_S = (e^{-a})^{\lambda}$

(on the other hand, if I is fixed then the limit is 1)

Due to the high complexity of the phenomenon, we are in a position to compute only worst case analysis

Let $\hat{G}_S$ = worst G_S, $\hat{G}_R$ = worst G_R

We show in Appendix B that

$$\hat{G}_S = \frac{1}{w}$$

$$\hat{G}_R = \frac{m}{\Omega} + \frac{1}{w}$$

Substituting these values in the expressions for G_R and G_S, we derive

$$\hat{q}_R = (1-\frac{1}{\Omega} - \frac{1}{mw})^{m-1} (1-\frac{1}{w})$$

$$\hat{q}_S = (1-\frac{1}{\Omega} - \frac{1}{mw})^{I+1}$$

Observe that as a consequence of the uniform traffic assumption made above,

$$\hat{q}_{R_i} = \hat{q}_R \text{ for all } i$$

from which it follows that

$$\hat{E}(A_i) = \hat{E}(A) \text{ for all } i$$

and

$$\hat{E}(B_i) = \hat{E}(B) \text{ for all } i$$

Naturally, not every repeater enters [leaves] the queue at the same time, but if the experiment were carried out a large number of times, on the average, R_i would enter [leave] the queue at E(A) [E(B)].

3. INITIALIZATION COMPUTATIONS

Consider ROP transmissions by R_i with probability of success q_R. Then A_i is a random variable geometrically distributed. Thus, $1/q_R$ represents the expected number of transmissions by R_i before the ROP reaches S. Since the ROP is put out by R_i every Ω slots (on the average).

$$\hat{E}(A_i) = \frac{\Omega}{q_R}$$

Consider label transmissions by S. $1/q_S$ represents the expected number of label transmissions by S. If we knew the cycle length at each retransmission, we could add these values to obtain the time necessary to deliver the label to R_i, such cycle length is unavailable, hence we must assume the worst case of mw. Thus, $\frac{mw}{q_S}$ is the time required to deliver one label to R_i. However, it takes $\frac{1}{q_R}$ such receptions and ETE transmissions before the station receives an ACK and eliminates this repeater from the queue.

Finally;

$$E(B_i) = \frac{\Omega}{q_R} + \frac{mw}{q_S} \quad \frac{1}{q_R} = \hat{\mathfrak{t}}$$

4. OPTIMIZATION OF INITIALIZATION TIME

It is clear that if Ω is small, the delay between ROP transmission is small, and this would apparently speed up initialization. However, if Ω is small, the traffic generated by the repeaters is large, causing q_S and q_R to decrease and thus slow down initialization.

On the other hand, if Ω is large, the traffic is small causing q_R and q_S to increase and thus speeding initialization; however, since the wait between ROP transmission is large, the initialization time may still increase.

The same holds true for w. It is apparent that optimal values of w and Ω must exist. From the previous section;

$$\mathfrak{T}(\Omega,w) = \frac{1}{[1 - (\frac{1}{\Omega} + \frac{1}{mw})]^{m-1}(1-\frac{1}{w})} \cdot \{\Omega + \frac{mw}{[1 - (\frac{1}{\Omega} + \frac{1}{mw})]^{I+1}}\}$$

Close form optimization of this function of two variables is unattainable. We must thus resort to numerical techniques, in particular point-to-point search, which works well in view of the limited domain of w and Ω (w and Ω must be integers since we are in a slotted environment).

The graphs of Figure 3 through Figure 8 show the trends.

Figure 3 depicts the behavior of $\hat{\mathfrak{T}}$ as a function of Ω and w. Basically, we see that the initialization time to the right of the optimum rises rather slowly, while to the left of the optimum raises drastically; hence, we should over estimate Ω^* rather than under estimate. Observe also that the station rate is much larger than the repeater rate; this is so since S must serve m repeaters.

Figure 4 shows the relation between $\Omega^*$and w at $\hat{\mathfrak{T}}^*$. It states that if the station transmits at a higher interval, so should the repeaters.

Figure 5 describes the behavior of $\hat{\mathrm{I}}^*$ as a function of w, pointing at the necessity of selecting the optimum value for w.

Figure 6 depicts the behavior of $\hat{\mathrm{I}}$ as a function of m. The graphs show that as m increases, the optimal initialization time increases approximately linearly with m (for fixed interference level I).

Figure 7 and Figure 8 illustrate the behavior of I as a function of I. The graphs show that as I increases, the optimal initialization time increases approximately linearly with I (for fixed m).

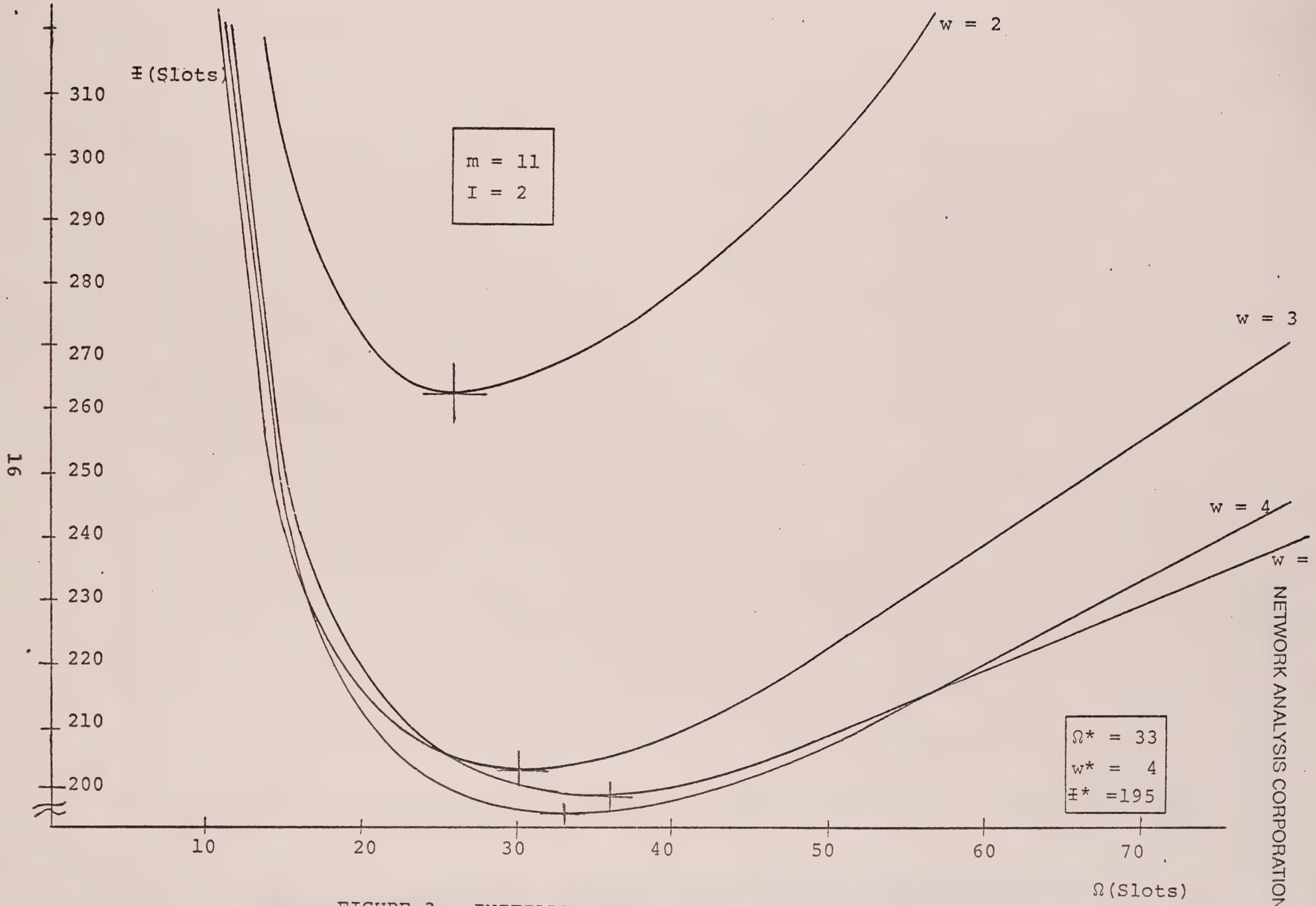

FIGURE 3: INITIALIZATION TIME AS FUNCTION OF Ω AND w

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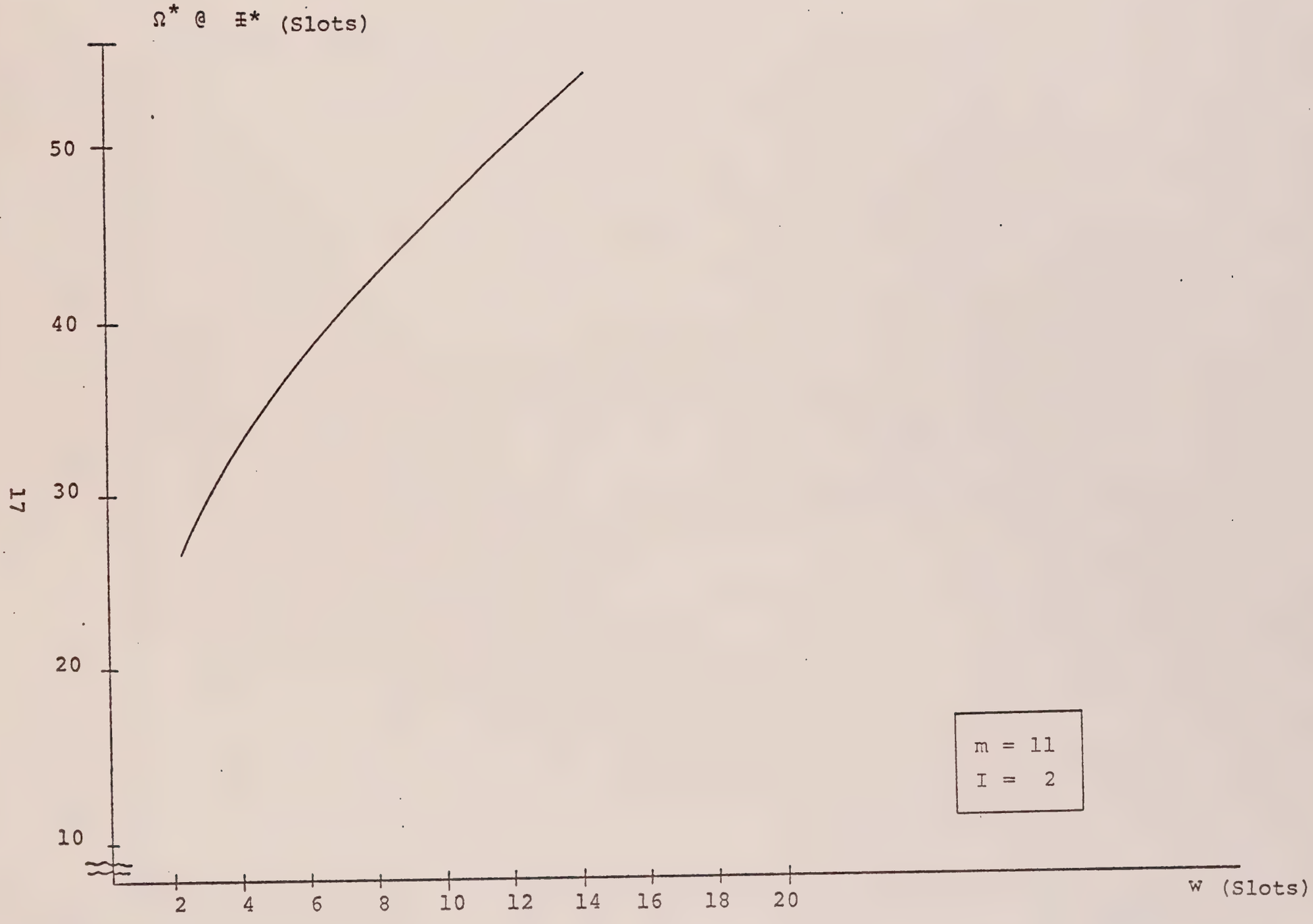

FIGURE 4: RELATION OF w AND Ω*, AT THE OPTIMUM INITIALIZATION TIME

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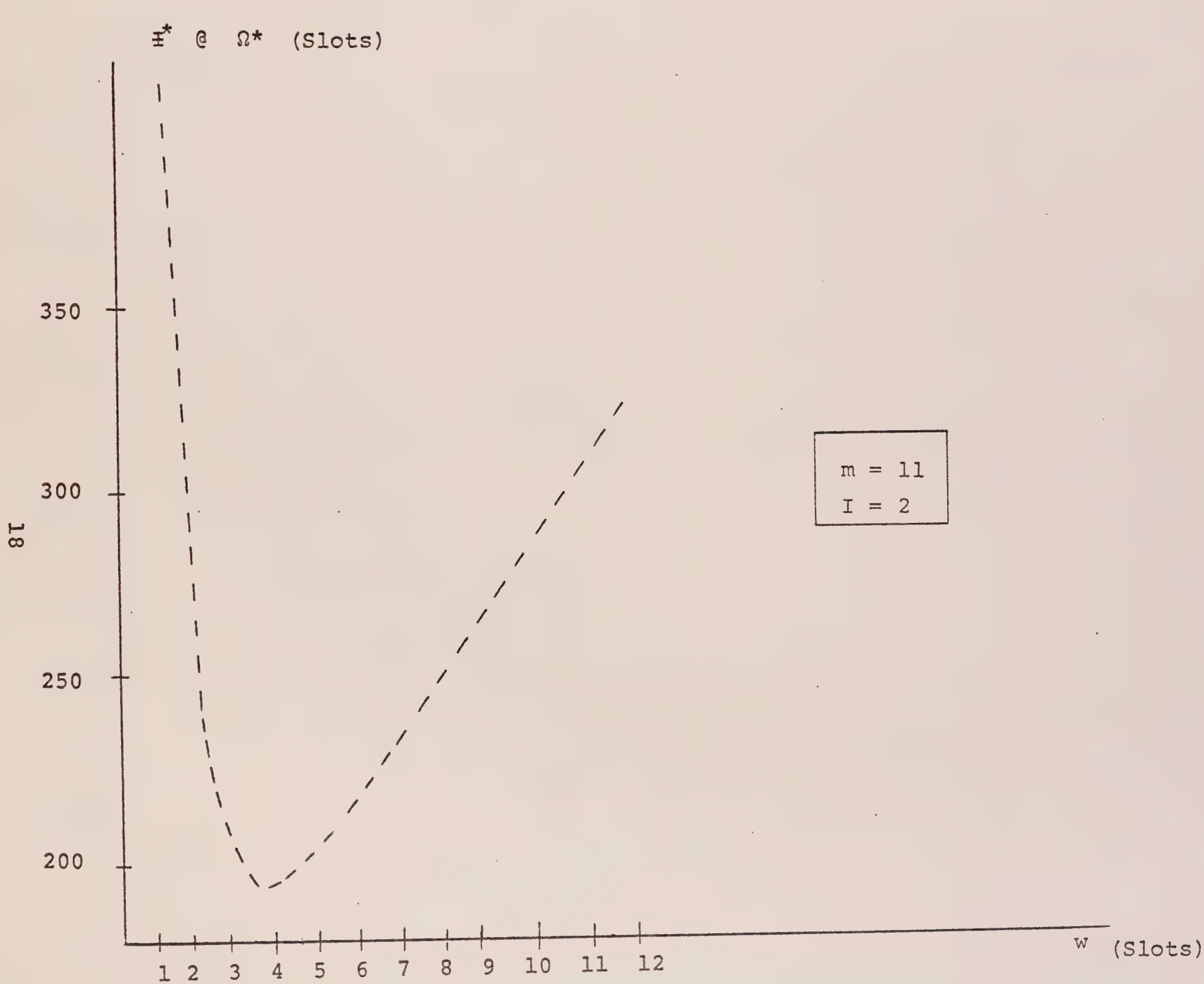

FIGURE 5: RELATION BETWEEN w AND Ŧ AT THE OPTIMUM REPEATER RATE

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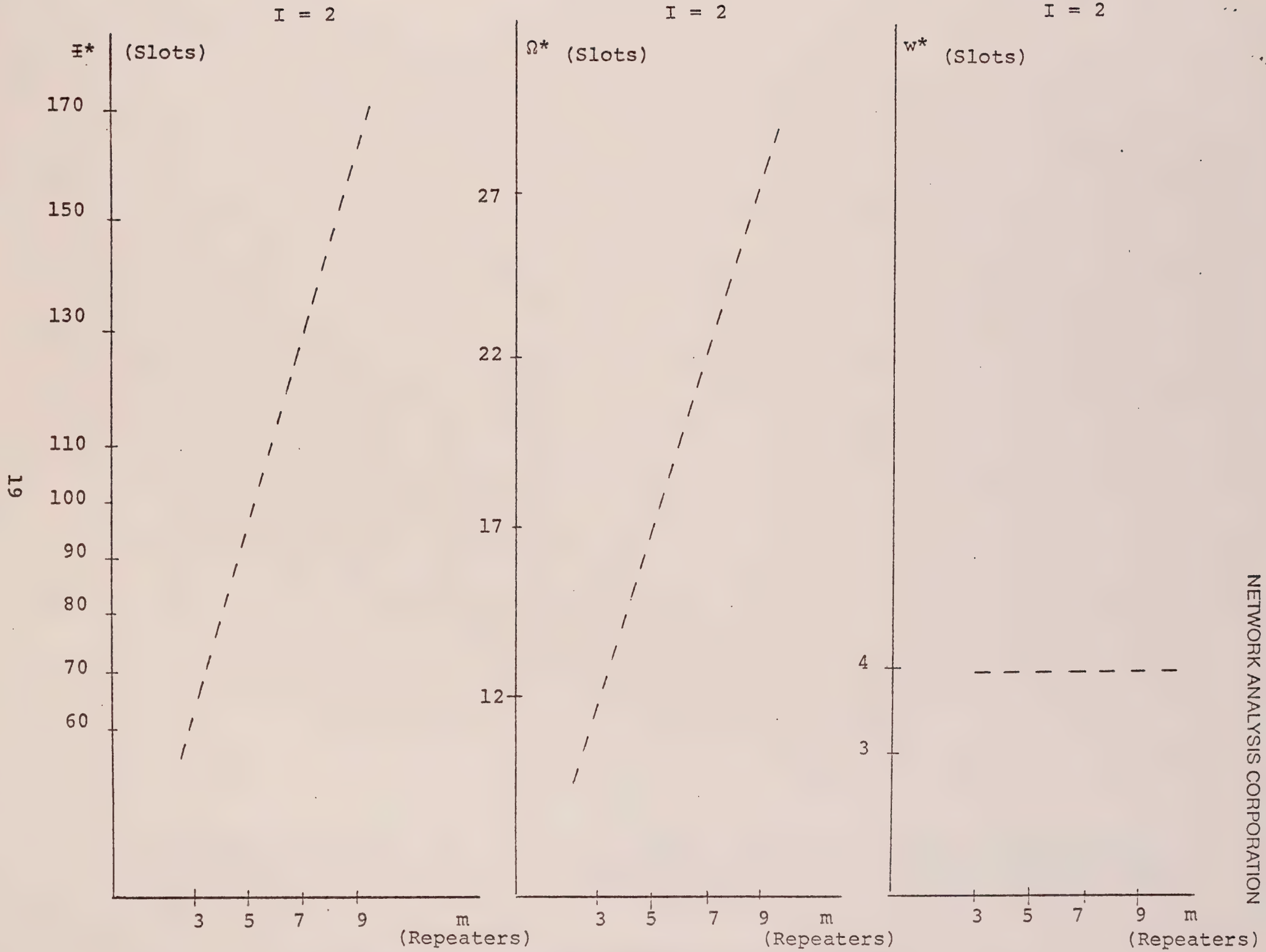

FIGURE 6: RELATION BETWEEN Ŧ*, Ω*, w* AND m

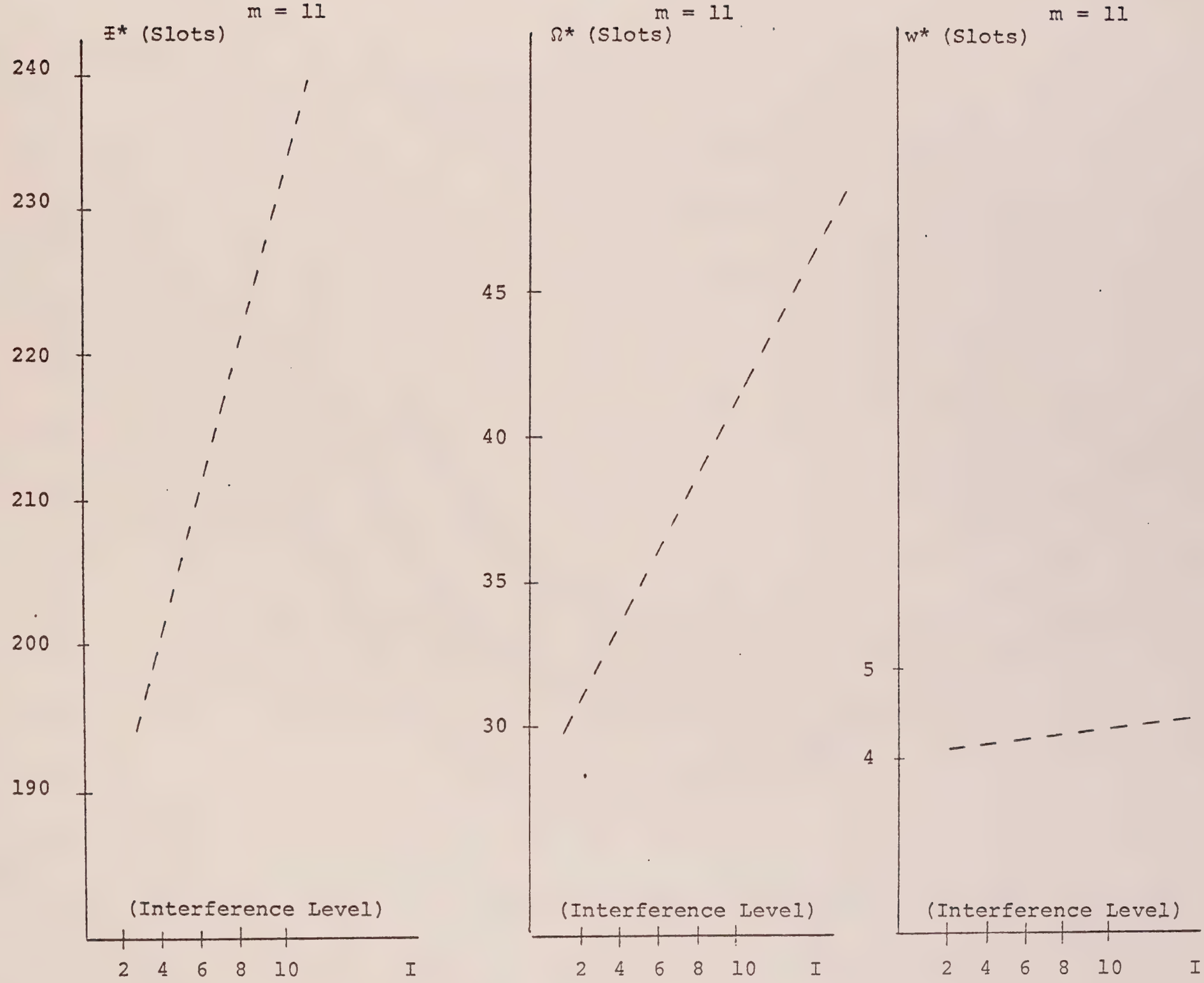

FIGURE 7: RELATION BETWEEN Ŧ*, Ω*, w* AND I

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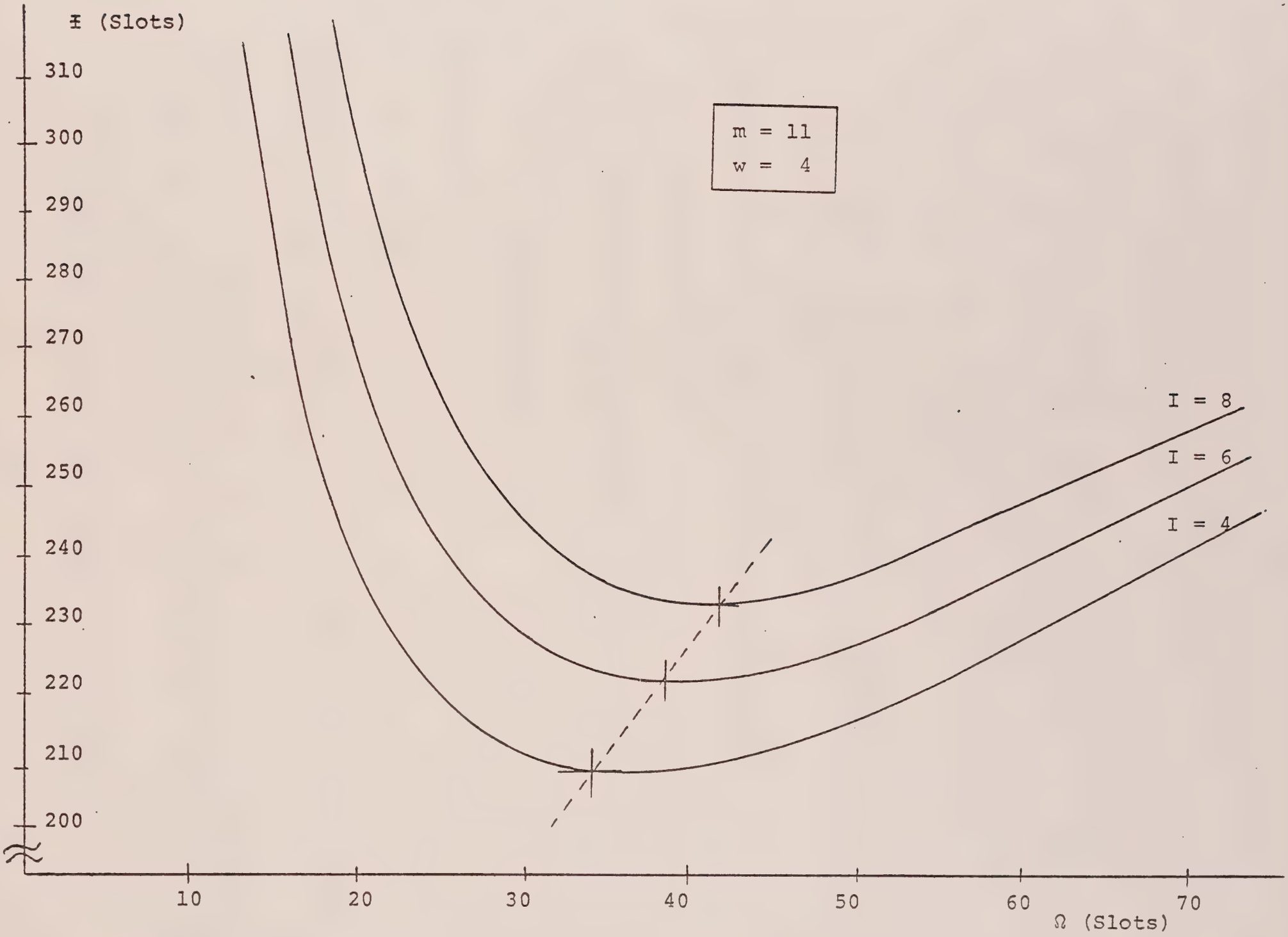

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FIGURE 8: RELATION BETWEEN Ŧ AND Ω AS A FUNCTION OF I

5. APPROXIMATE OPTIMAL VALUES

In lieu of a close form solution for the optimal values, we can carry out a satisfactory empirical evaluation of the optima.

Part A of Table 1 gives the optima for various values of m and I. Strikingly, $\hat{w}^*$ is constant over a large range of m and I. In fact $\hat{w}^* = 4$ in all cases, except when I is very close to m, in which case $\hat{w}^* = 5$. We thus conclude that:

$\hat{w}^*$ = optimal station rate, is nearly independent of m and I (except in case of extreme interference) and $w^* = 4$ slots.

Part B of the table gives the values for the quantity

$$h = m(1 + \frac{1}{m})^{m+I/2+1}$$

We see that $\hat{\Omega}^*$ is very close to this expression, we thus declare that

$$1. \quad \hat{\Omega}^* \simeq m(1+\frac{1}{m})^{m+I/2+1}$$

Observe I = 2,

$$2. \quad \hat{\Omega}^* \simeq me$$

where e = 2.718 ... (more generally, this holds also for the case where I is small and independent of m)

Actually, both approximations for $\hat{\Omega}^*$ tend to overstate the value of the optimum, as a comparison of Column B in Table 1, with column (d) reveals. We conclude that:

$\hat{\Omega}^*$ = optimal repeater rate, is an increasing function on m and I, and for small values of I, $\Omega^* \simeq me$.

A					B	C	D
(a) m	(b) I	(c) w*	(d) Ω*	(e) ∓*	Computed Ω* (rounded)	Computed ∓* (rounded)	Approxi-mation
3	2	4	12	63	12	75	
5	2	4	17	95	18	106	
7	2	4	22	128	23	138	
9	2	4	27	161	28	171	(2)
11	2	4	33	195	34	204	
50	2	4	133	842	140	858	
100	2	4	260	1672	275	1698	
1000	2	4	2566	16610	2722	16818	
10000	2	4	24626	165944	27186	168018	
100	10	4	273	1724	287	1750	
100	50	4	337	1986	350	2049	(1)
100	90	5	429	2250	427	2431	
11	4	4	36	208	37	219	
11	6	4	39	221	40	237	
11	8	4	42	235	44	256	(1)
11	10	5	49	247	48	278	

TABLE 1: NUMERICAL COMPARISON OF EXACT SOLUTION TO APPROXIMATE MODEL AND APPROXIMATE SOLUTION TO APPROXIMATE MODEL

Figure 9 compares the approximate optimal values with the actual optimal value using the results of Table 1. Observe the close fit for a large range of m.

With the above approximations, we can compute the worst case, approximate initialization time, as follows:

$$\hat{q}_R = (1 - (\frac{1}{\Omega} + \frac{1}{mw}))^{m-1}(1-\frac{1}{w})$$

$$\hat{q}_R^* \simeq [1 - (\frac{1}{me} + \frac{1}{4m})]^{m-1}(1-\frac{1}{4})$$

$$= \frac{3}{4}[1 - \frac{4+e}{4em}]^{m-1} = \frac{3}{4}[1 - \frac{4+e/4e}{m}]^{m-1}$$

$$\simeq \frac{3}{4} e^{4+e/4e} \simeq .40$$

Similarly,

$$\hat{q}_S^* \simeq [1 - (\frac{1}{me} - \frac{1}{4m})]^{I+1} = [1 - \frac{4+e/4e}{m}]^{m\frac{I+1}{m}}$$

$$\simeq (e^{4+e/4e})^{(I+1)/m} \simeq (.54)^{(I+1)/m}$$

Finally,

$$\hat{\mathfrak{T}}^* = \frac{\Omega}{q_R} + \frac{mw}{q_R q_S} \simeq \frac{me}{.40} + \frac{4m}{.4(.54)^{(I+1)/m}}$$

$$= 6.79m + \frac{10m}{(.54)^{(I+1)/m}}$$

Part C of Table 1 shows for comparison the computed values of $\hat{\mathfrak{T}}^*$.

So far $\mathfrak{T}$ has been quoted in a number of slots in the ALOHA procedure. Typical value for slot length for an ROP is 5×10^{-3} seconds. Hence, for example, in a 100 repeater system at interference level 2, it would take 0.84 seconds to initialize a typical repeater.

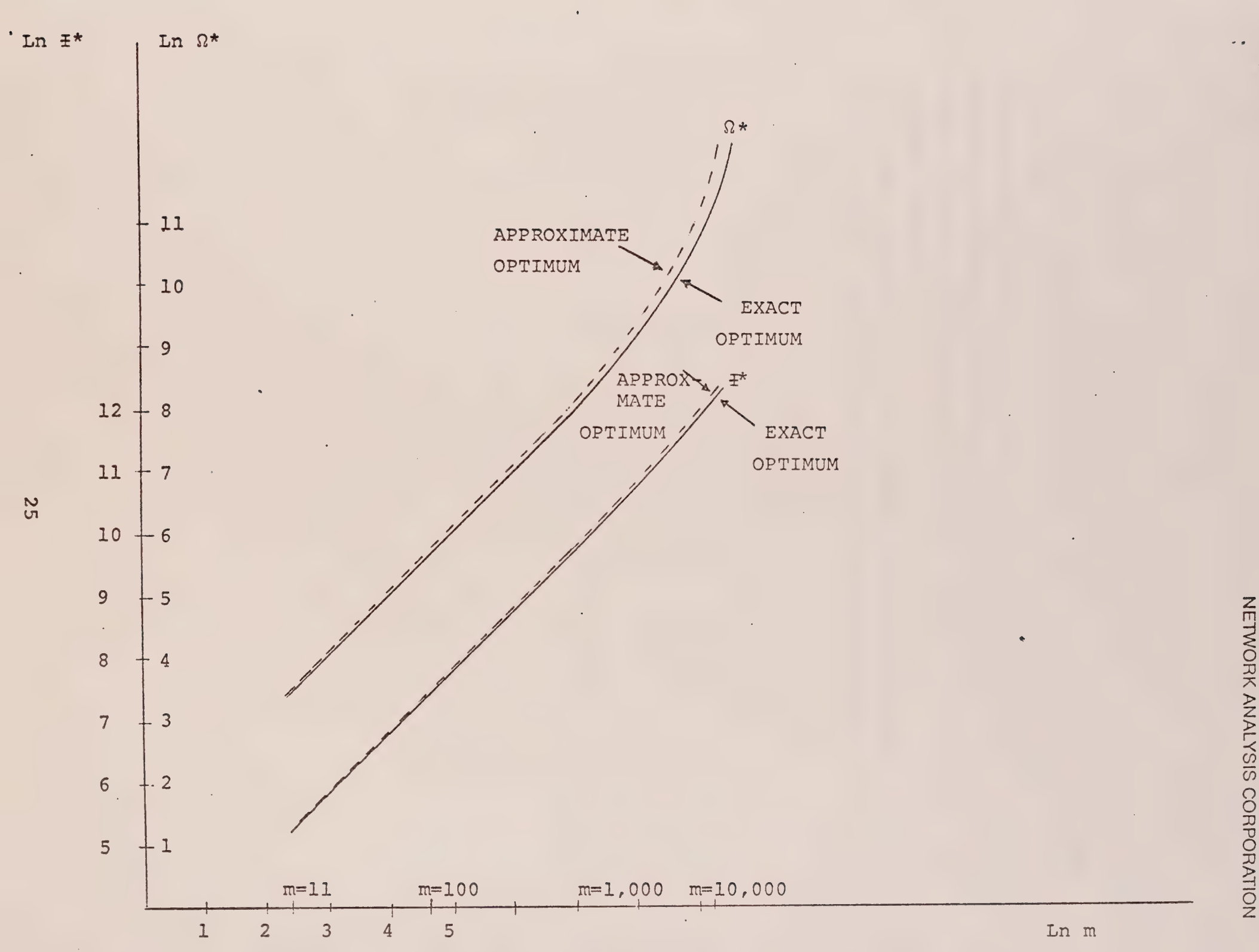

FIGURE 9: COMPARISON OF EXACT SOLUTION TO APPROXIMATE MODEL AND APPROXIMATE SOLUTION TO APPROXIMATE MODEL

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6. CONCLUSION

In this note a worst case study for initialization has been conducted; it is believed that the same trends in respect to the dependency of $\bar{I}$ on I and m, etc., hold true in the actual initialization procedure. To ascertain this an exact model via a Markov chain is being developed, and will be compared to the model described presently.

Other approximate models (multiple retransmissions, finite buffer size, 2-hops) will follow in a subsequent note.

APPENDIX A

We wish to show that the traffic generated by the Active ACK is of the order 2m packets.

Consider a given R_i, which is in the process of transmitting ROP's. By assumption, this repeater broadcasts such packets up to time

$$\frac{\Omega}{q_R} + \frac{mw}{q_S} \simeq \frac{\Omega}{q_R} + \frac{4m}{q_S}$$

(on the average at the optimum).
Thus, the average number of ROP transmissions is

$$\frac{1}{q_R} + \frac{4m}{q_S \Omega} = \frac{1}{q_R} + \frac{4m}{q_S em} = \frac{1}{q_R} + \frac{4}{e}\frac{1}{q_S} = \eta$$

Consider η as a fixed parameter.
Let N be the number of copies of the ROP which are successfully received by S. N is binomially distributed $B(q_R,\eta)$ (granted fixed η).

$$E(N) = \eta q_R = 1 + \frac{4}{e}\frac{q_R}{q_S}$$

Globally, for all repeaters

$$m\ (1+\frac{4}{e}\ \frac{q_R}{q_S}) = \gamma$$

ROP's reach S. Now

$$\gamma = m(1+\frac{4}{e}[1 - (\frac{1}{\Omega} + \frac{1}{mw})]^{m-I-2}\ \frac{3}{4})$$

$$< m(1+\frac{3}{e}) = m\ (1+1.1) = 2.1m$$

Hence, (on the average, of the optimum, approximately, as worse case) 2.1m Active ACK need be broadcast; this is small compared to the total packet traffic.

APPENDIX B

In this Appendix, we compute the busiest rate (or worst case) for repeaters and stations.

It is clear that

$$\hat{G}_S = \text{Worst } G_S = \frac{1}{w}, \quad 0 \leq t \leq \mp$$

The determination of the worst G_R is more intricate. Consider some reference interval R (see Figure B1). In this interval, if no label has been received by R_j, we would have a rate of

$$\frac{R/\Omega}{R}$$

packets issued by the repeater under consideration; if, however, w_j labels had been received by R_j, then w_j ETE ACK would be issued, with a corresponding loss of i_j ROP packets, (where i_j is a function of the instance when the first label was received); hence, a typical rate of packet generation by R_j is

$$\gamma_j = \frac{R/\Omega + w_j - i_j}{R}$$

Globally, $G_R = \sum_j \gamma_j = \frac{m}{\Omega} + \frac{\Sigma w_j}{R} - \frac{\Sigma i_j}{R}$

However, $\Sigma w_j = R/w$, thus

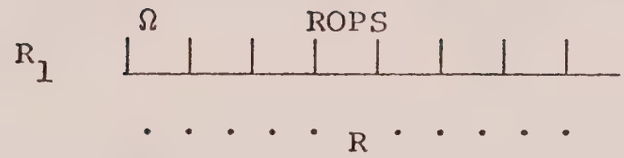
$$G_R = \frac{m}{\Omega} + \frac{1}{w} - \frac{\Sigma i_j}{R}$$

Very little can be done with the i_j's, notwithstanding this, it is clear that

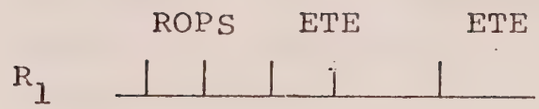
$$\hat{G}_R = \text{worst } G_R = \frac{m}{\Omega} + \frac{1}{w},$$

from which

$$\frac{\hat{G}_R}{m} = \frac{1}{\Omega} + \frac{1}{mw}$$



(a) Rate for $R_1 = \frac{8+0-0}{R}$



(b) Rate for $R_1 = \frac{8+2-5}{R}$

Observation period of 8Ω slots. In (a) no labels are received; the net rate is given above. In (b) two labels are received at $t_1 = 4\Omega$ and $t_2 = 6\Omega$ hence, R_1 issues 8-5 = 3 ROPs on $(0, 4\Omega)$ and two ETE ACKs at $t = 4\Omega$ and $t = 6\Omega$. The net rate is given above.

FIGURE B1: DETERMINATION OF $\hat{G}_R$

APPENDIX C

1. Determination of $\mathcal{J}$

In this Appendix we show how to derive (at least formally)

$$\mathcal{J} = E\left(\max_{1 \le i \le m} B_i\right)$$

Clearly, for the most case,

$$\hat{B}_i = \Omega \tilde{R}_i + mw \tilde{\tilde{R}}_i \tilde{S}$$

where $\tilde{R}_i$, $\tilde{\tilde{R}}_i$, $\tilde{S}$ are the number of ROP transmissions by repeater R_i, ETE transmission by repeater R_i and station transmissions respectively, to obtain successful packet delivery.
Now,

$$\text{Prob}(\tilde{R}_i = v) = (1 - q_R)^{v-1} q_R = \text{Prob}(\tilde{\tilde{R}}_i = v)$$

$$\text{Prob}(\tilde{S} = u) = (1 - q_S)^{u-1} q_S$$

with u, v integers

Thus,

$$\text{Prob}\{B_i = b\} = \sum_{\substack{\text{all integer} \\ \text{solutions } u,z,v \\ \text{to} \\ v\Omega + mwzu = b}} (1 - q_R)^{v-1} q_R (1 - q_R)^{z-1} q_R (1 - q_S)^{u-1} q_S$$

with b integer. In general, if a specific answer is desired, we must result to numerical techniques.

$$\text{Let } Y_m = \max_{1 \le i \le m} B_i$$

then, from standard statistics

$$\text{Prob}(Y_m=y) = m\ \text{Prob}(B_i=y)[\text{Prob}(B_i \leq y)]^{m-1}$$

(any i can be used since the B_i's are assumed to be independent and identically distributed).

Hence,

$$\mathcal{J} = E(Y_m) = \sum_{y=1}^{\infty} m \cdot y \cdot (\text{Prob}(B_i=y)\ [\text{Prob}(B_i \leq y)]^{m-1}$$

where certain integer values are taken on with probability zero. Closed form solutions are elusive, but numerical computations are applicable.

2. COMPROMISE

To bypass the complexity discussed above we apply Chebyshev's inequality.

We desire Π such that,

$$\text{Prob}(B_i \leq \Pi) \geq \alpha$$

where $\alpha = .95$ or some other convenient value. For the random variable X, Chebyshev's inequality states that for any c

$$\text{Prob}(|x-c| < \varepsilon) \geq 1 - \frac{1}{\varepsilon^2} E(x-c)^2$$

Let $c = o$; thus,

$$\text{Prob}(x < \varepsilon) \geq 1 - \frac{1}{\varepsilon^2} E[(x)^2]$$

Since,

for the particular combination of $\tilde{R} = v$, $\tilde{R} = z$, $\tilde{S} = u$

$$\text{Prob}[B_i = v\Omega + mwzu] = (1-q_R)^{v-1}(1-q_R)^{z-1}(1-q_S)^{u-1} q_R^2 q_S$$

we obtain

$$E(B_i^2) = \sum_{u=1}^{\infty} \sum_{v=1}^{\infty} \sum_{z=1}^{\infty} (v\Omega + mwzu)^2 (1-q_R)^{v-1}(1-q_R)^{z-1}(1-q_S)^{u-1} q_S q_R^2$$

After considerable algebraic manipulation

$$E(B_i^2) = q = \Omega^2 \frac{2+q_R}{q_R^2} + 2\Omega mw \frac{1}{q_R^2 q_S} + m^2 w^2 \frac{(2+q_R)(2+q_S)}{q_R^2 q_S^2}$$

Hence, if

$$\text{Prob}(B_i \leq \Pi) \geq 1 - \frac{1}{\Pi^2} q = \alpha$$

then it must be that

$$\Pi = \sqrt{\frac{q}{1-\alpha}} = (1-\alpha)^{-1/2}\sqrt{q}$$

Numerical calculations can now be carried out to obtain actual values for Π.
Clearly,

$$\mathfrak{T} \leq \Pi$$

The limited investigation which has been carried out is shown in Table C1, where a comparison is shown between Π at confidence level 0.95 and $\mathfrak{T}$.
Observe that the Π-curve achieves the optimum in a neighborhood of the optimum of $\mathfrak{T}$; in fact it can be seen that for I=2,

$$\Omega^*_{\Pi} \simeq me^{1.2}$$

$$w^*_{\Pi} = 3$$

indicating that at least for what concerns the optimal repeater and station rate, we need not be too concerned with what measure for initialization we choose to consider.
Also, we can see that

$$\Pi \simeq 7.97\,\mathfrak{T} = 1.78(1-\alpha)^{-1/2}\,\mathfrak{T},$$

indicating the increase in time required to be able to say, with high confidence, that a typical repeater is initialized.

		$\alpha = .95$					
m	I	w*	Ω*	$\mp$*	w*	Ω*	$\amalg$*
11	2	4	33	195	3	39	1,557
50	2	4	133	842	3	159	6,707
100	2	4	260	1,672	3	315	13,313
1000	2	4	2,566	16,610	3	3,097	132,228
10000	2	4	24,626	165,944	3	30,920	1,321,379

TABLE C1

COMPARISON BETWEEN $\mp$, $\amalg$

3. CONCLUSION

The following has been addressed in this Appendix. Assuming that the random variables B_i are independent and identically distributed, and in addition, assuming worst cases, we have indicated how to compute

$$\text{Prob}\{\max B_i \leq t\}$$

(and in particular $\mathcal{J} = E(\max B_i)$)

This was shown to be rather intractable. The following heuristic procedure has then been suggested:

For any α compute t_Π, such that,

$$\text{Prob}\{B_i \leq t_\Pi\} \geq \alpha,$$

for any repeater R_i.

Since by this time R_i (for any i) is initialized with high probability (granted α close to 1) then every repeater is likely to be initialized. Thus, we say heuristically that for high α,

$$\mathcal{J} \approx t_\Pi$$